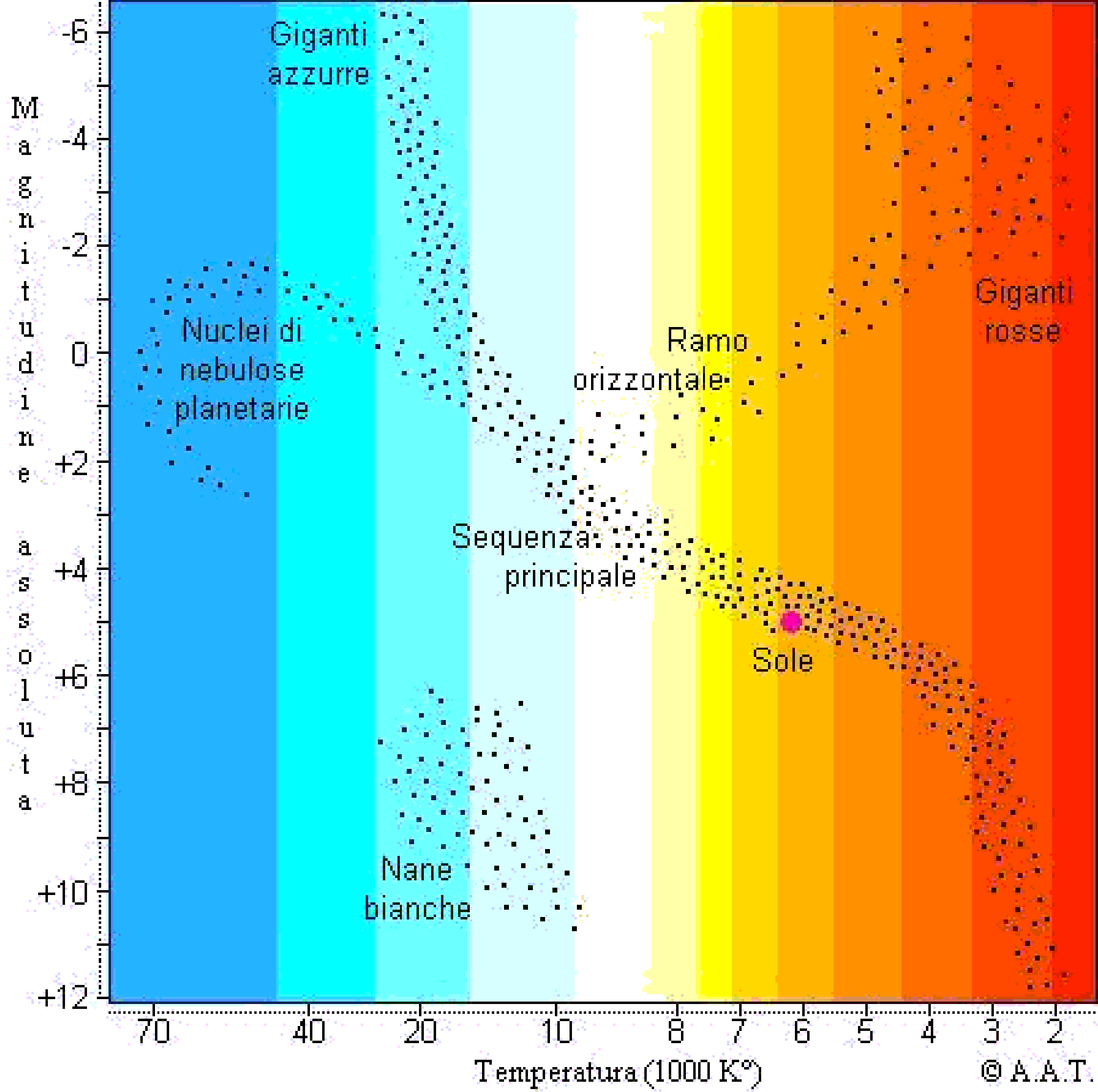
Collapse supernovae

- Combustion stages at the end of main sequence
- Toward collapse
- Fate of a massive star: neutron star or black hole?
- Why is neutronization convenient?
- Emitted energy in neutron star formation
- Release of gravitational energy: neutrino role
- Estimate of neutrino flux from a SN at the Galaxy center
- The case of SN 1987A

The path in HR diagram

- From a first look at the H-R diagram one can immediately observe how stars tend to arrange in distinct regions:
- The main evolution structure is the diagonal that goes from top left (where stars are more massive, hotter and brighter) to the lower right corner (where stars are lighter, cooler and less bright), called main sequence.
- In lower left region is located the white dwarf sequence, while, above main sequence to the right, there are red giants and supergiants.



The **main sequence** is an evolution structure in the Hertzsprung-Russell diagram that identifies the stage in which stars produce energy by converting hydrogen into helium in the core, via nuclear fusion reaction.

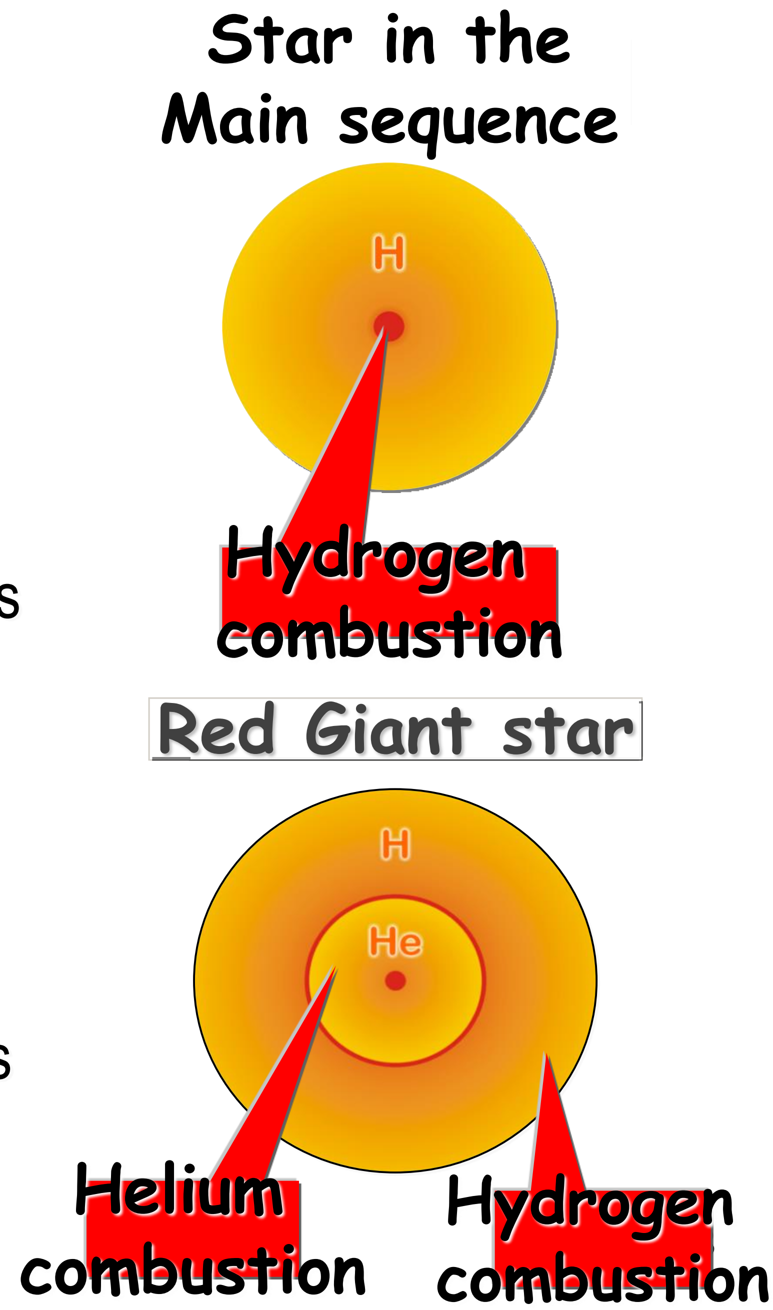
From yellow dwarfs to red giants

•Stars move along the HR plane reaching different condition of L and T related to their path in nuclear combustion.

•Along themain sequence, stars burn H in the center until it is exhausted.

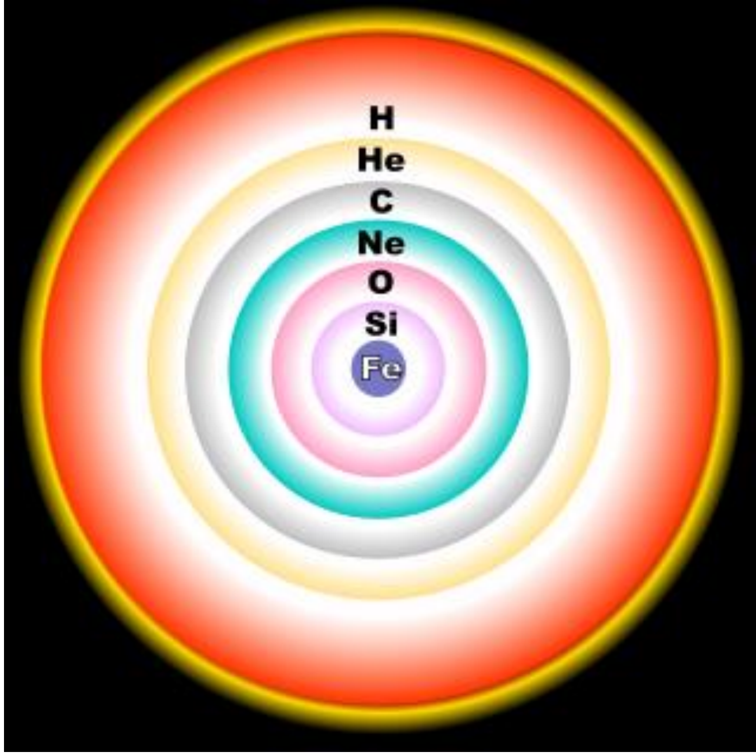
•When H in the center is finished, the core contracts due to effect of gravitation. In this way it reaches the required temperature and density to transform Helium into Carbon, in the center, via three body reaction: 3 ^{4}He -> ^{12} C.

•Exterior of the star begins to burn hydrogen and its envelope is inflated to form a “red giant”.



The fate of heavier stars

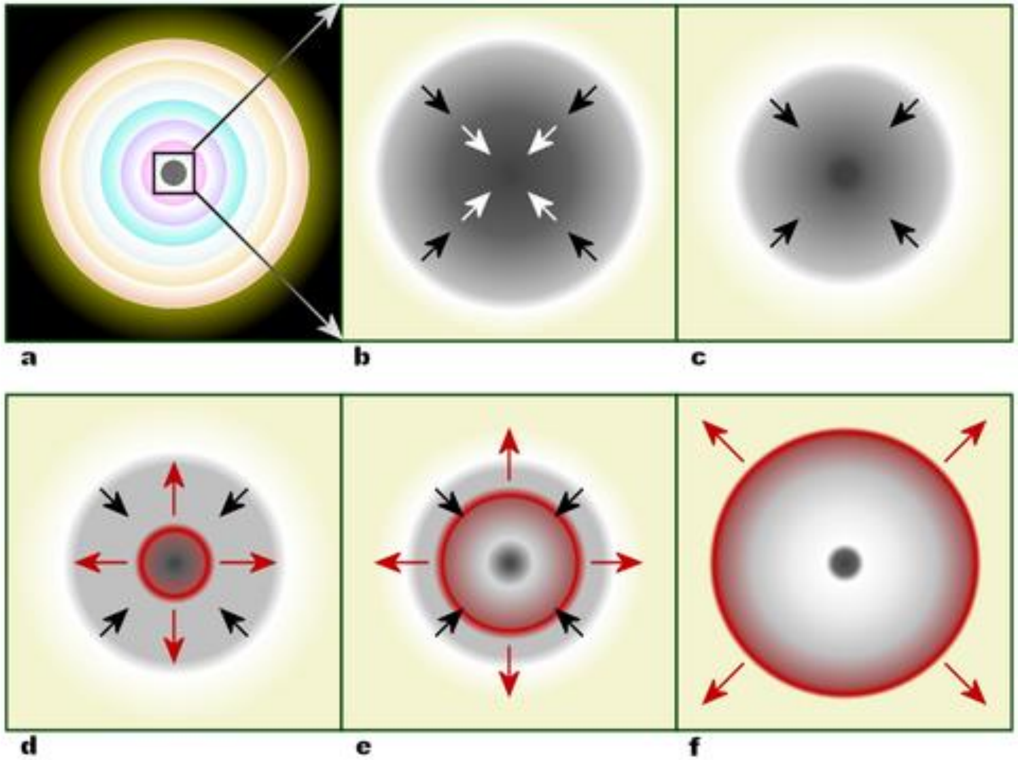
- In case of heavy enough stars, the process proceeds, generating a shell structure in which reactions are gradually triggered between nuclei with increasing Z.*
- In this way, Carbon is converted into Oxygen, and then the Oxygen into Silicon, up to Iron.
- The combustion of heavier elements happens faster and faster. The table shows the burning time scale for a 15 M_o star.
- The maximum binding energy is reached with formation of the Iron core, so there are no more exo-energetic reation which can sustain the star.



Reaction	Timescale
Hydrogen	10 million years
Helium	1 million years
Carbon	300 years
Oxygen	200 days
Silicon	2 days

*In stars with mass comparable to the solar mass, the gas pressure of degenerate electrons stabilizes the star, which no longer needs to contract. It becomes a white dwarf, a crystal of carbon and/or hydrogen, see Appendix

Implosion and explosion

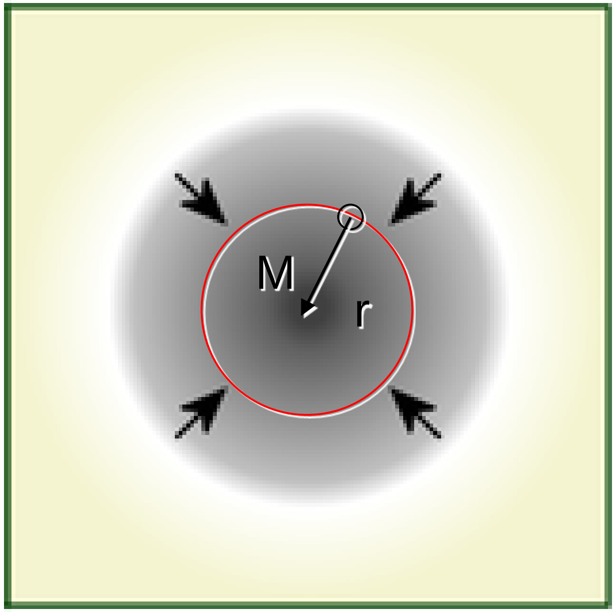


The core, no longer sustained by nuclear reactions, implodes while surrounding matter falls around it. When the core reaches nuclear densities the contraction stops and matter bounces back and finally the star explodes generating a "collapse supernova" *, an object appearing greatly bright, approximately 10^5 stars a few hours after its appearence.

*It should be noted that so far the process of explosion is not quantitatively understood.

**Supernovae are indicated with the year of appearence followed by a letter which shows the order in the year.

Time scale of gravitational collapse: $t \approx (G\rho)^{-1/2}$



•If hydrostatic equilibrium is broken and matter is in free fall, a body at a distance r from the center falls with an acceleration $a=-GM/r^2$ where M is the mass contained within a radius r, which remains constant during the collapse.

•If initial speed was negligible and original radius large enough, integrating the equation one has $v^2= 2GM/r$, which can be still integrated. The elapsed time to reach r=0 is: $t= 2/3\ (r^3/2GM)^{1/2}$. Expression in brackets can be expressed in terms of density ρ getting $t =(\pi/6)^{1/2} (G\rho)^{-1/2}$

•Equation $t \approx (G\rho)^{-1/2}$ is the typical time scale of gravitational collapse*.

•If Sun ($\rho \approx 1.5\ 10^3$ kg/m^3) was not supported by gas pressure, it should collapse in a time of order **3000 s**.

•For nuclear densities, $\rho \approx (1/4\ m_p/fm^3) \approx 0.4\ 10^{18}$ kg/m^3, typical times are of order **ms.**

*Expressing the density in unity of 1012g/cm3, eq. is written:

$$\mathbf{t_{coll}} \approx \frac{1}{\sqrt{\mathbf{G\rho}}} \approx \frac{\mathbf{4\,ms}}{\sqrt{\boldsymbol{\rho}_{12}}}$$

Collapse supernovae and neutron stars

Walter Baade (1893-1960)

Fritz Zwicky (1898-1974)

Baade and Zwicky were the first ones to think of a possible connection between supernovae explosions and neutron stars formation [Phys. Rev. 45 (1934) 138]

What happens in the core?

- Gravity tends to squeeze the core, which can no longer be supported by the pressure generated by exothermic nuclear reaction energy.
- Quantum mechanics "opposes" gravity with a pressure associated to typical quantum effects:
- Heisenberg principle: according to quantum mechanics, the simple fact to confine particles in a sphere of radius r implies that they have a momentum (Heisenberg): **$p \geq \hbar/r$**. At this momentum corresponds a pressure *
- Pauli principle: a maximum of 2 identical fermions can be in the same phase space cell. This means that if the 2 fermions with lower energy have each momentum $p = \hbar/r$, the following will have $p = 2\,\hbar/r$ and so on, so that average momenta carried by particles are greater than if they were all in the ground state; this creates a pressure that grows more than linearly with the particles number.
- We will estimate **equilibrium conditions** assuming to have only one kind of particles, neutrons; then we will see why this hypothesis corresponds to a minimum energy situation.
- *Exercise: determine energy U of a particle in the fundamental state of a radius r box, and then determine the pressure on the walls from $dU=PdV$

Neutron gas in gravitational field

- Suppose to have N neutrons in a radius R sphere; the total volume is $V \approx R^3$ and the volume available for each pair of particles* is $v=V/(N/2) \approx R^3/N$, i.e. each particle is confined within a linear dimension $r \approx R/N^{1/3}$ and therefore Heisenberg principle will give a momentum at least:

$$p = \hbar/r = \hbar N^{1/3}/R$$

- It follows that its energy** is:

$$\varepsilon = (p^2c^2+m^2c^4)^{1/2} = [\,(\hbar c)^2 N^{2/3}/R^2) + m^2c^4\,]^{1/2}$$

- Each neutron will be attracted by other neutrons with an energy $U \approx -Gm^2 N/R$; then total energy per particle is $E = \varepsilon + U$, i.e.

$$\mathbf{E = [(\hbar c)^2 N^{2/3}/R^2 + m^2c^4\,]^{1/2} - Gm^2 N/R}$$

* You will notice that here we are applying the esclusion principle

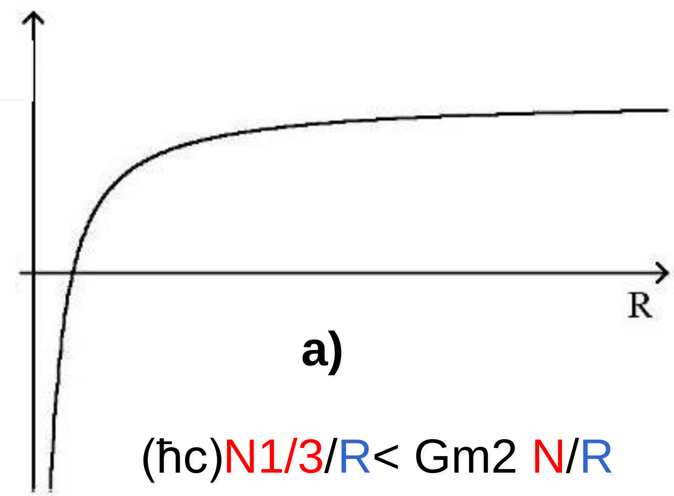
** In the state of minumum energy, which we are concerned, the momentum is given in the previous expression.

Stationary conditions

$$E(N,R) = [(\hbar c)^2 N^{2/3}/R^2 + m^2c^4]^{1/2} - Gm^2 N/R$$

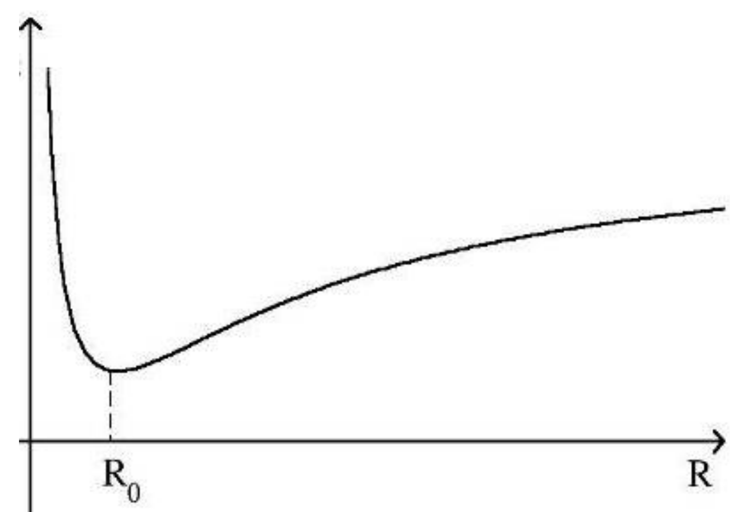
- The particle energy equation allows us to establish, for a fixed N, if there is an equilibrium condition (i.e. minimum energy) and to determine the radius of equilibrium.
- For large R the term obviously becomes: $E = mc^2 - Gm^2 N/R$, i.e. energy decreases with decreasing distance.
- For small R the term $\mathbf{m^2c^4}$ under square root will be negligible and therefore: $\mathbf{E(N,R) \approx (\hbar c)N^{1/3}/R - Gm^2 N/R}$
- Note that both terms behave as 1/R and hence there are two possible cases:
  - **a)** there is no equilibrium configuration; thestar contracts indefinitely becoming a black hole.
  - **b)** there is a stable structure with radius Ro (neutron star)

E(N,R)



a)

(ħc)N1/3/R< Gm2 N/R



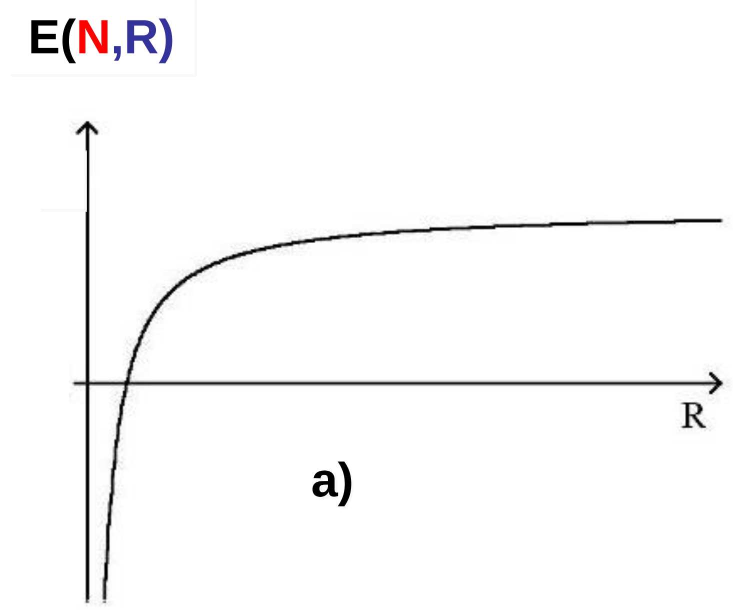
b)

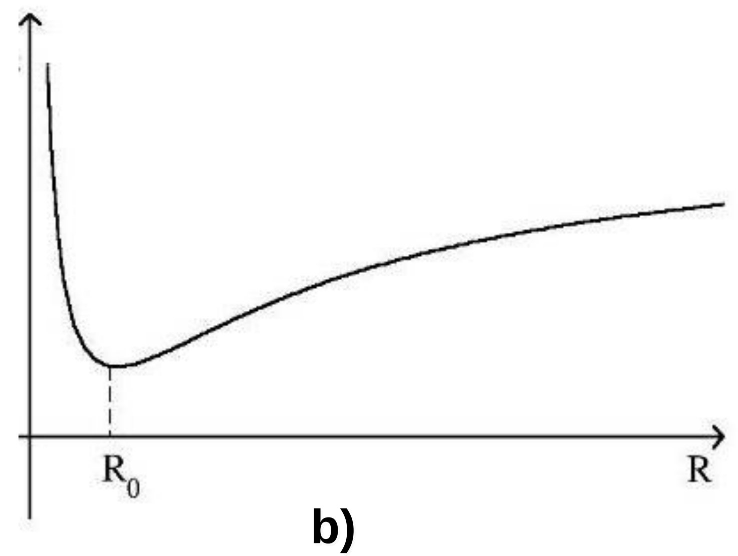
(ħc)N1/3/R> Gm2 N/R

Chandrasekar mass

$$E(N,R) = [(\hbar c)^2 N^{2/3}/R^2 + m^2c^4]^{1/2} - Gm^2 N/R$$

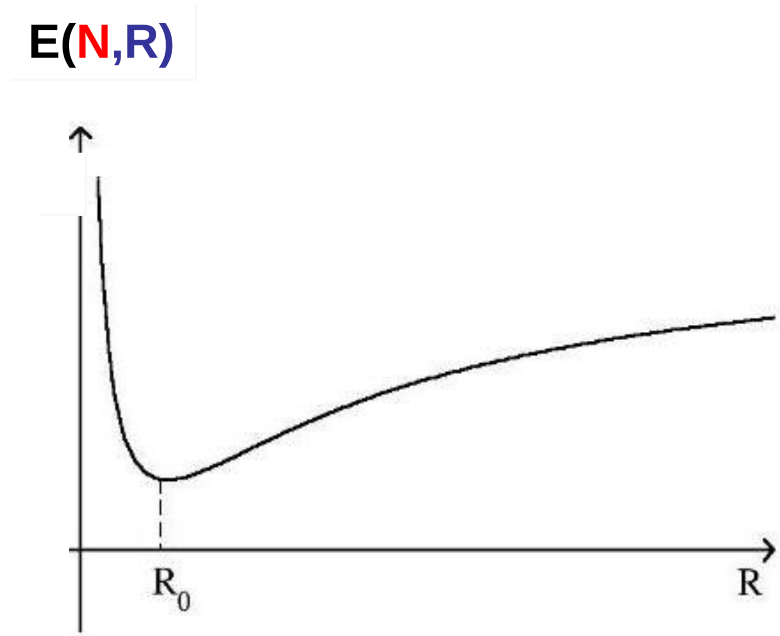
- The condition that separates cases a) and b) comes from $(\hbar c)N^{1/3}/R = Gm^2 N/R$ i.e. $N = [\hbar c/ Gm^2]^{3/2}$
- Remember that $\alpha_G = Gm^2/\hbar c = 6\ 10^{-39}$ is the equivalent of the fine structure constant for gravitation, i.e. the dimensionless constant that characterizes the gravitational interaction between two nucleons.
- It follows that $N \approx (10^{38})^{3/2} = 10^{57}$, that is a mass $M = mN \approx M_o$, i.e. a mass of order of that of the Sun.
- A more accurate estimate shows that the mass limit (Chandrasekar mass) is 1.5 M_o
- Therefore, for values up to the mass limit we have stable systems, of which we can study properties such as radius, density and binding energy.





$$E(N,R) = [(\hbar c)^2 N^{2/3}/R^2 + m^2c^4]^{1/2} - Gm^2 N/R$$

Neutron star radius



- For a star at equilibrium, the R_o is determined by **dE/dR = 0**
- We can assume that, around the equilibrium condition, neutrons are non-relativistic (we will check this assumption later). In this case:

$$E(N,R) = mc^2 + p^2/2m - Gm^2 N/R$$
$$= mc^2 + \hbar^2 N^{2/3}/(2mR^2) - Gm^2 N/R$$

•Requiring dE/dR=0 one has $\hbar^2 N^{2/3}/(mR_0^3) = Gm^2 N/R_0^2$ i.e.

$$\mathbf{R_0 = \hbar^2/Gm^3\, N^{-1/3} = N^{-1/3}(\hbar c^2/Gm^2)(\hbar c/m) = N^{-1/3}\,\alpha_G^{-1}\lambda_c.}$$

where we recognize the role of α_G and Compton length of nucleon λ_c. .

•Putting $N=10^{57}$ one has R_o=10km, i.e. a star like our Sun squeezed in a **10 km radius**.

•At this point one can verify that non-relativistic approximation is satisfied in the sense that neutron momentum is (exercise):

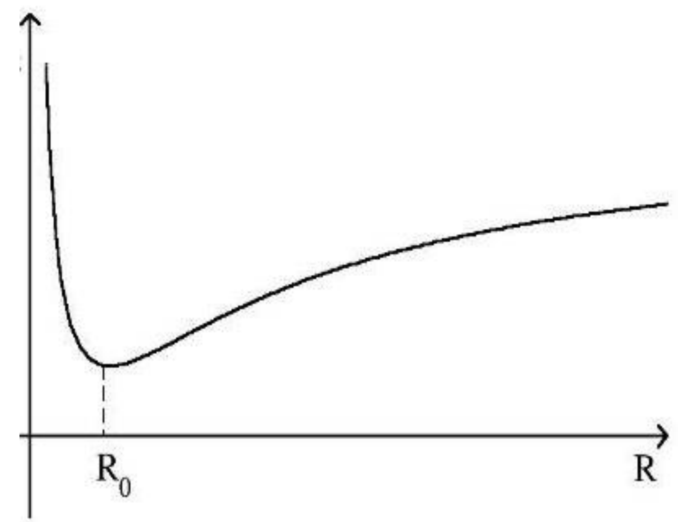
• $pc \approx 200\text{MeV} << mc^2$.

Neutron star density

$$E(N,R) = [(\hbar c)^2 N^{2/3}/R^2 + m^2c^4]^{1/2} - Gm^2 N/R$$

- For the considered case ($N=10^{57}$, R_o=10km), the number of neutrons per unit volume is $n= N/(4/3\ \pi R_o^3) = 0.25\ fm^{-3}$.
- This is comparable with nuclear matter density. The neutron star is essentially a huge nucleus, with so many neutrons as there are in the sun
- Note that we have masses comparable to that of Sun and linear dimensions smaller by a factor of 10^5 . Densities are thus 10^{15} times greater, i.e. of order $10^{15} g/cm^3$.
- A white dwarf is a star with mass comparable to that of the Sun and supported by the pressure of degenerate electron gas*. In this case the radius of the stable configuration is about 6000 km (≈ Earth radius) and therefore the densities are of order $10^6\ g/cm^3$.

E(N,R)



*Exercise: determine the stability condition of a star supported by degenerate electrons gas pressure

Why a neutron star?

•In the collapsing star temperatures are high, corresponding to photons with energy of order MeV, so that nuclei are dissociated and e, p and n are present in comparable number.

•As we have seen, fermions typical momentum (either electrons, protons or neutrons) are:

$$p_i \sim \hbar N_i^{1/3}/R \sim 200\ \mathrm{MeV}/c$$

where it has been assumed that for each particle type $N_i \sim 10^{57}$.

•Typical energies are then:

$$T_p \sim T_n \sim p^2/2m \sim 20\ \mathrm{MeV}$$ **NON-RELATIVISTIC**

$$E_e \sim pc \sim 200\ \mathrm{MeV}$$ **RELATIVISTIC**

•So electrons colliding with protons have sufficient energy to give:

$$e^- + p \longrightarrow n + \nu_e$$

• The produced neutrinos (which have weak interactions) pass through the star escaping freely, so that the reverse reaction does not occur. The entire star turns into neutrons (NEUTRONIZATION)

Why neutrons are stable?

• Free neutrons decay through beta process

$$n \longrightarrow p + e^- + \text{anti-}\nu_e$$

• This process is possible because Q= M_n-M_p-M_e =0.782 MeV >0

•The produced electrons have kinetic energy of order MeV, with a maximum momentum p_{max} =(Q^2 +2Qme) ^{½}= 1.1 MeV/c

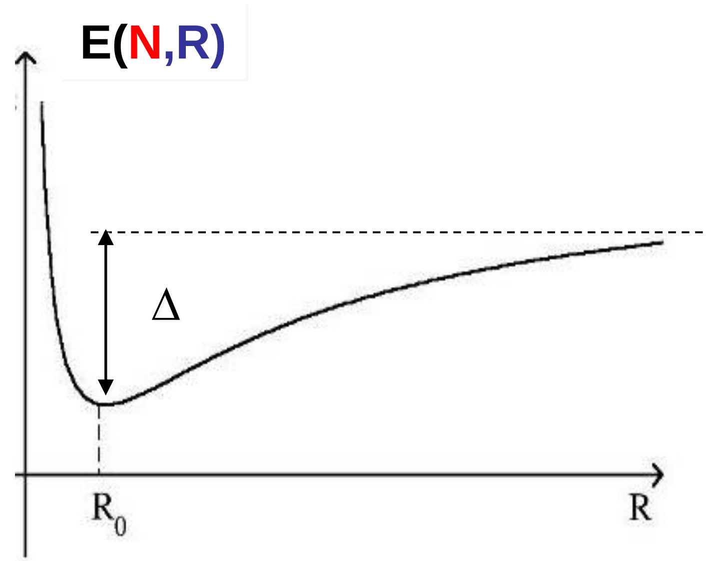
•Electrons accumulate in the star, and so the Fermi momentum $p_e = \hbar N_e^{1/3}/R$ increases. Decay stops when the Fermi momentum is equal to p_{max}.

•This means that the number of produced electrons is $N_e = (p_{max} R/\hbar)^3$ whose ratio to the number of neutrons N is

$$Y = N_e/N = [p_{max} R/\hbar N^{1/3}]^3 = [p_{max}/p_n]^3 \approx 10^{-7}$$

In other words, after the decay of a small fraction of neutrons, the rest are stable.

Energy released in neutron star formation



•For each neutron, the energy gained in the formation of the neutron star is $\Delta = mc^2 - E(N,R_o)$

•As non-relativistic approximation is valid, one has;

$$\Delta = p^2/2m - Gm^2N/R = T - U$$

•It's easy to see that at R=Ro one has T= ½ U and therefore

$$\Delta = ½\, U = ½\, Gm^2N/Ro\ .$$

•This is valid for each neutron and therefore the total released energy is $E_b = N\,\Delta$ i.e.

$$E_b \approx 1/2 \frac{G_N M^2}{R_o} = 10^{46}\,\mathrm{J} \left(\frac{M}{M_{\mathrm{sole}}}\right)^2 \left(\frac{10\ \mathrm{Km}}{R}\right)$$

•To appreciate the meaning of this amount of energy, ≈10^{46} J, note that the luminosity of Sun is 4 10^{26} W, so this is the energy that the Sun would radiate in 10^{12} years.

•It also corresponds to the energy radiated by the Galaxy in about 30 years.

Neutrino production

•During collapse the star is "hot", with particle kinetic energies of ~ 10-100 MeV.

• Under these conditions there is a whole series of reactions that can produce neutrinos:

- Neutronization:

• $e^- + p \longrightarrow n + \nu_e$ (electronic capture from free neutrons)

• $e^- + (Z,A) \longrightarrow \nu_e + (Z-1,A)$ (electron capture from nuclei)

- Pair production:

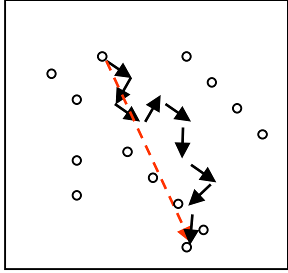
• $e^+ + e^- \longrightarrow \nu + \text{anti-}\nu$ (pair annihilation)

• $e- + \gamma \longrightarrow e- + \nu + \text{anti } \nu$ (photo-annihilation)

• $e- + (Z,A) \longrightarrow (Z,A) + e- + \nu + \text{anti}\nu$ (bremsstrahlung)

•Note that in the reactions of neutronization only electron neutrinos are produced, while in the process of pair production both neutrinos and antineutrinos are produced in any family.

Matter opacity for crossing neutrinos



• Usually, but not always, neutrinos can pass through great distances without collisions. In extremely dense matter, the processes of absorption and scattering prevent neutrinos from escaping freely from the collapasing core and sorrounding matter.

• Is is worth mentioning that density in a neutron star is of the order

$$n \approx 10^{57} / (4\ 10^{18} cm^3) \approx 2.5\ 10^{38} / cm^{-3}$$

• Scattering on free nucleons and heavy nuclei is the main source of opacity for neutrinos:

•Elastic scattering on nucleons, ν + n —> ν + n, has cross section:

$$\sigma \approx 10^{-44}\ cm^2\ (E/\ 1MeV)^2$$

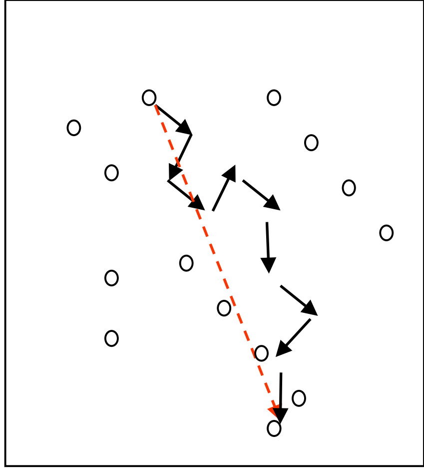
•Coherent elastic scattering on heavy nuclei, ν + (Z,A) —> ν + (Z,A), has cross section

$$\sigma \approx 10^{-44} cm^2\ A^2 (E/\ 1MeV)^2$$

•System temperature are of order $T \approx 4\ 10^{10}$ K ⇔ $kT \approx$ 3 MeV and neutrinos are produced with energies of order of tens MeV. Neutrinos with these energies have $\sigma \approx 10^{-40} cm^2$ and therefore mean free path of order

$$\lambda = 1/(n\sigma) \approx 1/[(10^{-40}\ cm^2)(2.5\ 10^{38} cm^{-3})] \quad \rightarrow \lambda \approx 40\ cm$$

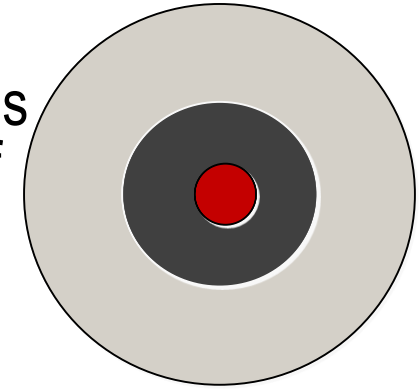
Neutrino diffusion in a neutron proto-star



- Let us estimate the time required for neutrinos to emerge from a neutron star with radius $r \approx 10$ km
- Since mean free path satisfies $\lambda << r$ neutrinos make multiple collisions before they can get out of the star.
- Brownian motion law tells us that the average distance traveled in a time t is given by

$$<R^2> = \lambda\, v\, t$$

- Placing $<R^2> = r^2$ and keeping in mind that for neutrinos v=c one obtains: $t \approx r^2/\lambda v \approx 1$ s
- It should be also noted that out of the neutron star, matter is still very dense and opaque to neutrinos. The neutrino-sphere, that is the region after last scattering surface, is located at distances of about 100 km from the center. Ultimately, the time required for neutrinos to get out is of the order 10 seconds,



$$t_{dif} \approx 10 \text{ s}$$

- Matter is still much more opaque to radiation: energy is therefore almost entirely carried by neutrinos, and only a fraction much less than 1/100 is in the form of e.m. radiation.

Neutrinos emission

•Neutrinos and antineutrinos thus carry almost all the energy emitted in the gravitational collapse. The emitted power is therefore:

$$W = E_b/t_{dif} \approx 10^{52} \text{ erg/s}$$

•Energy is carried by neutrinos with average energy ε= 10 MeV.
It follows that the luminosity of neutrinos is

$$L= dN_\nu/dt= W/ \varepsilon \approx 10^{57} \text{ s}^{-1} .$$

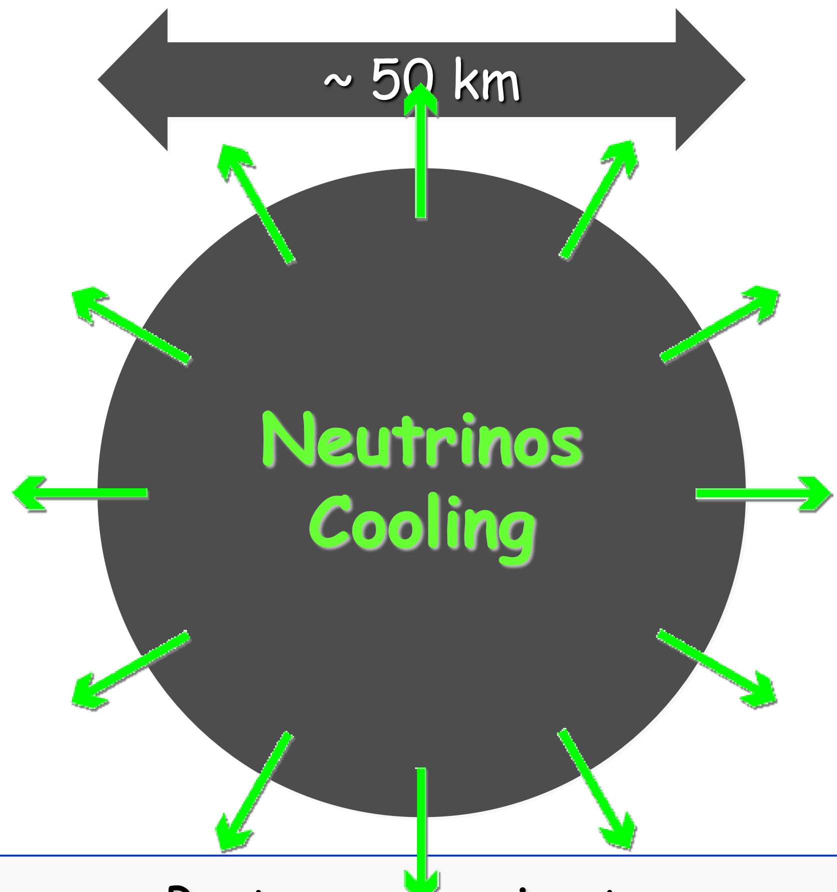
•The flux at a distance D will be therefore Φ= L/(4π D^2) ; placing D=10kpc one obtains for a supernova at the center of Galaxy

$$\Phi \approx 10^{11} \text{ cm}^{-2} \text{ s}^{-1}$$

•This flux is stronger than that of the Sun, for a very short time.
•Note that this flux is distribuited uniformely among neutrinos and antineutrinos of each
•In particular, there is a flux of about 10^{10} cm^{-2} s^{-1} **electron antineutrinos**, (remind that the Sun emits neutrinos).

Summary: Supernova explosions and stellar collapse

"Newborn" neutron stars



Proto-neutronic star

$\rho \approx \rho_{nuc} = 10^{15}$ g cm^{-3}

$T \approx 20$ MeV

Gravitational binding energy:

$E_b \approx 10^{53}$ erg $\approx$ 10% $M_0 c^2$

This is emitted as:

99% Neutrinos

1% kinetic energy

0,01% E.m. radiation

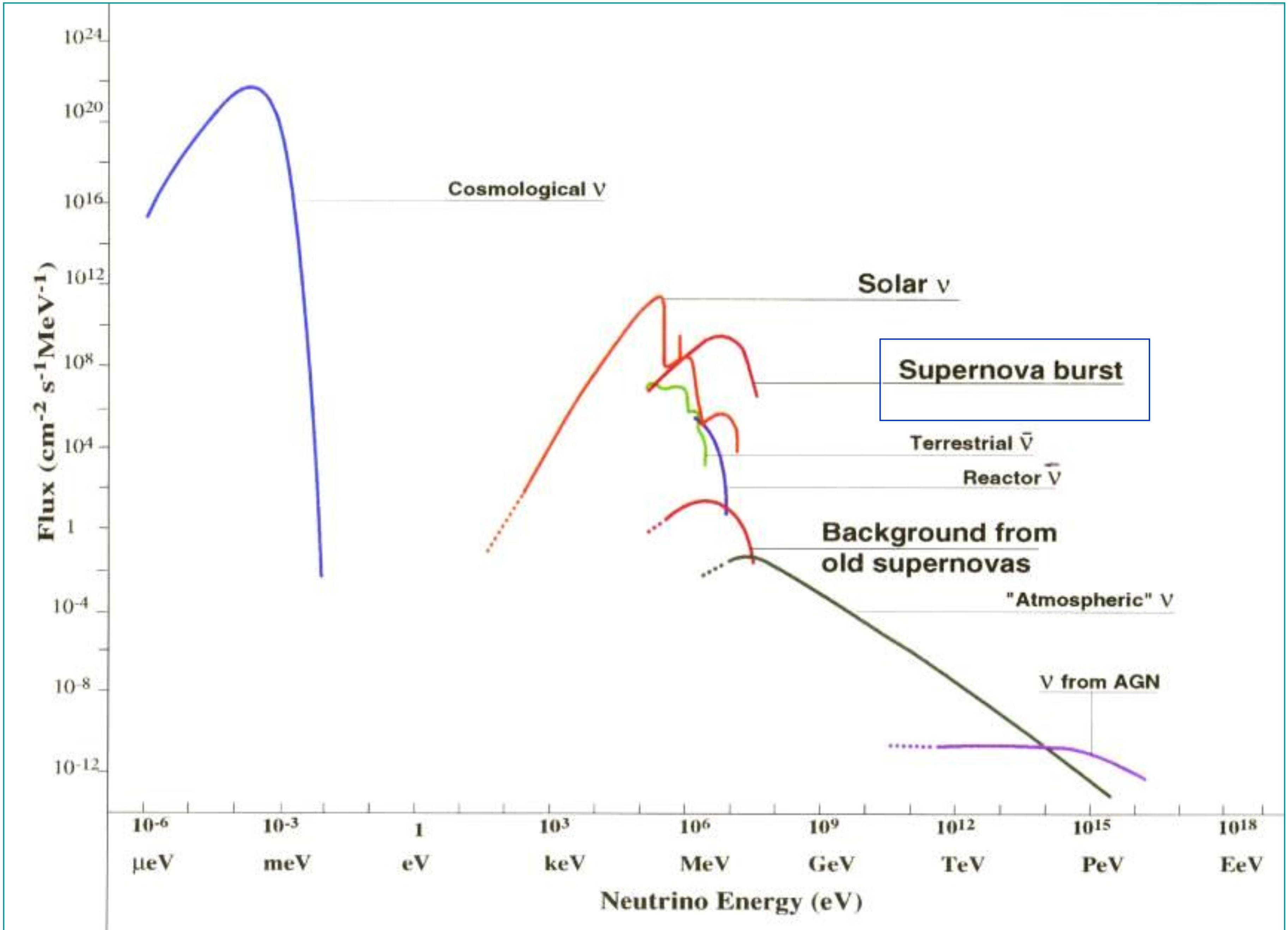
Neutrinos luminosity:

$L_\nu \approx 10^{52}$ erg / sec

$\approx 10^{19} L_0$

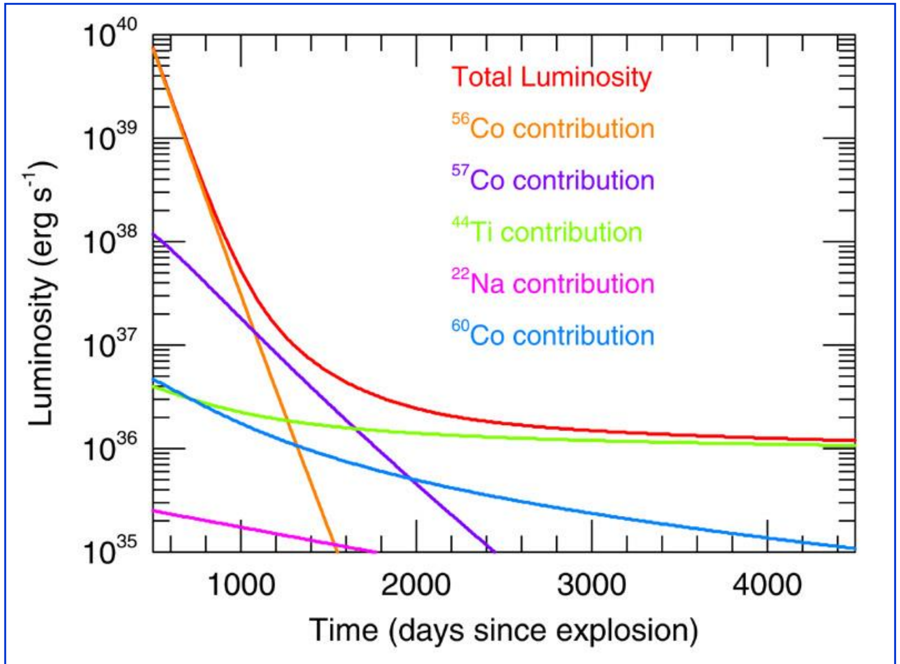
While it lasts, it shines more than the entire visible universe.

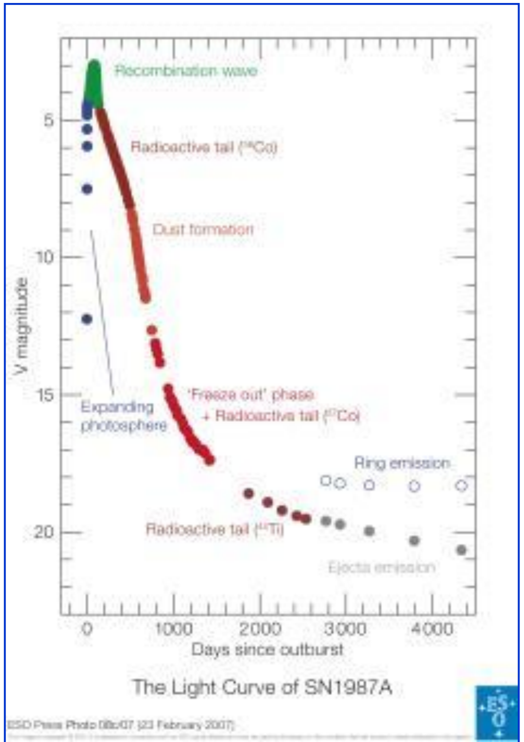
Supernova burst from the galaxy center compared to other natural sources of neutrinos



SN1987A supernova

- In February 1987 there was a SN in the Large Magellan Cloud*, a small satellite galaxy of Milky Way at a distance of about 50 kpc from us.
- For the first time, (anti)neutrinos have been detected from a Supernova, starting the extra-solar neutrino astronomy
- Results and implications of these observations will be discussed in the final chapter





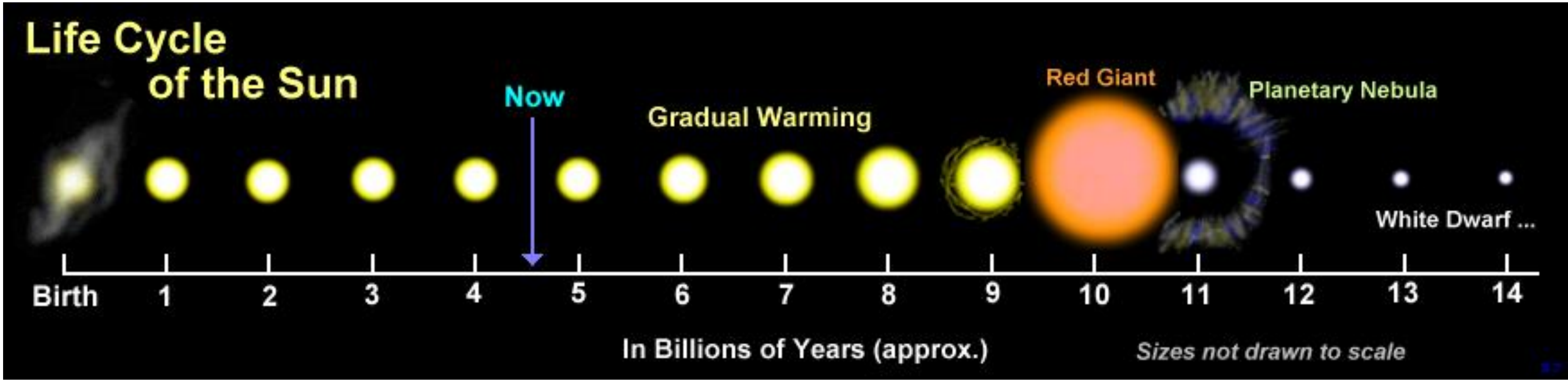
How many supernovae in the universe?

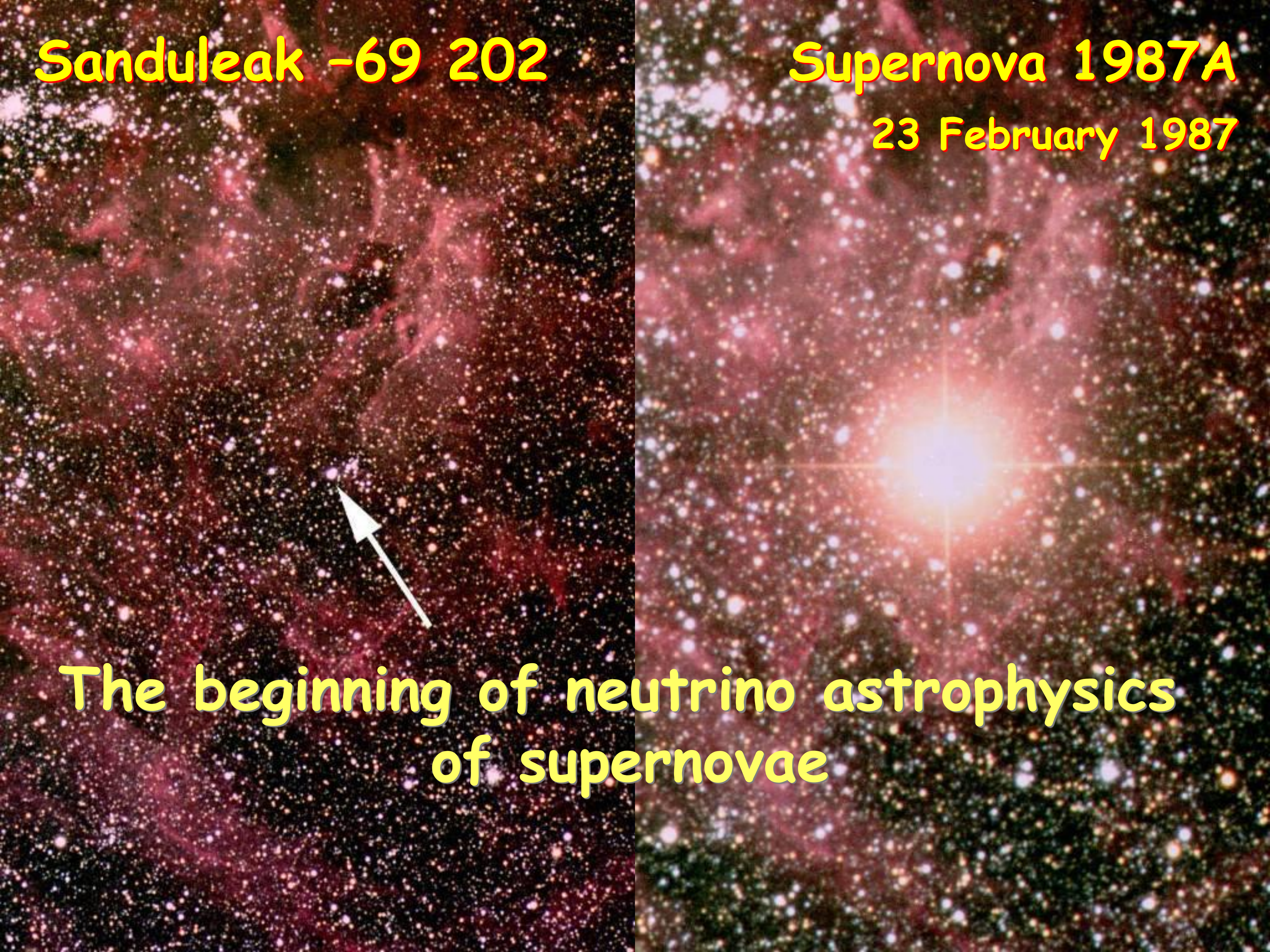
• SN 1987A in LMC (Large Magellan cloud) is the only with certain identification of a progenitor star.
• In our galaxy, in historical times approximately equally were observed.
•It is believed that in a galaxy like our there are about two supernovae per century
•The observed number is mush smaller because:
i) civilization developed in the northern hemisphere, where one cannot see the galactic center, which should provides explosions
ii) dust obscures a large part of sky, at least to the naked eye
•The observable universe contains about 10^{11} galaxies; if the supernova rate is the same as in our own, then in a year there are 10^9 SN

Appendix:

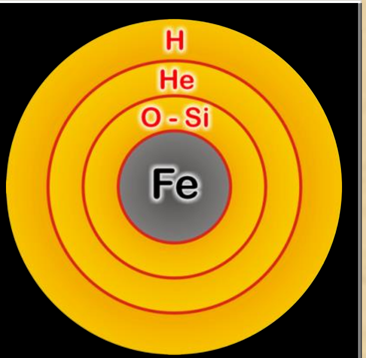
- Sun's fate
- Sandoulek and SN 1987A
- SN I and collapse SN

Sun’s fate


Life Cycle
of the Sun
Now
Gradual Warming
Red Giant
Planetary Nebula
White Dwarf ...
Birth
1
2
3
4
5
6
7
8
9
10
11
12
13
14
In Billions of Years (approx.)
Sizes not drawn to scale


Sanduleak -69 202
Supernova 1987A
23 February 1987
The beginning of neutrino astrophysics
of supernovae

Ia type vs. Core-Collapse Supernovae

Tipo Ia	Core collapse (Tipo II, Ib/c)
White dwarf carbon-oxygen (remainings of a littele star) increases materia from a companion 	Degenerate iron core of a massive star increases area by surface nuclear combustion 

Chandrasekhar limit reached - $M_{Ch} \approx 1.5\ M_O$

COLLAPSE OCCURS IN

nuclear combustion of C and O → nuclear deflagration ("fusion bomb" Triggered by collapse)	Collapse to nuclear densities Implosion → Explosion
Powered by nuclear energy	Powered by gravitation
Gain of nuclear energy ~ 1 MeV per nucleon	Gain of gravitational energy ~ 100 MeV per nucleon 99% in neutrinos

"Visibile" released energy comparable to ~ 3×10^{51} erg